

July, 2000

OU-HET 353

hep-ph/0007066

Quantum effects for the neutrino mixing matrix in the democratic-type model

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Abstract

We investigate the quantum effects for the democratic-type neutrino mass matrix given at the right-handed neutrino mass scale m_R in order to see (i) whether $\theta_{23} = -\pi/4$ predicted by the model is stable to explain the atmospheric neutrino anomaly, (ii) how θ_{12} and θ_{13} behave, and (iii) whether the predicted Dirac CP phase δ keeps maximal size, at the weak scale m_Z . We find that, for the (inversely) hierarchical mass spectrum with $m_1 \sim m_2$, θ_{23} and θ_{13} are stable, while θ_{12} is not so, which leads to the possibility that the solar neutrino mixing angle can become large at m_Z even if it is taken small at m_R . We also show that δ keeps almost maximal for the above mass spectrum, and our model can give the large CP violation effect in the future neutrino oscillation experiments if the solar neutrino puzzle is explained by the large mixing angle MSW solution.

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1 Introduction

Recent neutrino experiments have been strengthening the evidence of the neutrino mixing [1, 2]. The study of the neutrino mixing opens a new phase for our deeper understanding of neutrino physics.

Let us summarize the present experimental data on the neutrino mixing. From the recent analysis of the atmospheric neutrino anomaly, we have the following allowed regions of the mixing angle and the mass squared difference as [1]

$$\sin^2 2\theta_{\text{atm}} = 0.85 \sim 1, \quad \Delta m_{\text{atm}}^2 = 2 \times 10^{-3} \sim 6 \times 10^{-3} \text{ (eV}^2\text{)}. \quad (1)$$

The oscillation interpretation for the solar neutrino problem has still several parameter choices as [2, 3]¹

- The large mixing angle (LMA) MSW solution

$$\sin^2 2\theta_{\text{LMA}} = 0.5 \sim 1, \quad \Delta m_{\text{LMA}}^2 = 1 \times 10^{-5} \sim 1 \times 10^{-4} \text{ (eV}^2\text{)}, \quad (2)$$

- The small mixing angle (SMA) MSW solution

$$\sin^2 2\theta_{\text{SMA}} = 10^{-3} \sim 2 \times 10^{-2}, \quad \Delta m_{\text{SMA}}^2 = 4 \times 10^{-6} \sim 10^{-5} \text{ (eV}^2\text{)}, \quad (3)$$

- The vacuum oscillation (VO) solution

$$\sin^2 2\theta_{\text{VO}} = 0.75 \sim 1, \quad \Delta m_{\text{VO}}^2 = 10^{-11} \sim 10^{-10} \text{ (eV}^2\text{)}. \quad (4)$$

In our earlier paper [4, 5], we proposed a democratic-type mass matrix for left-handed Majorana neutrinos. This model has quite special predictions, $\theta_{23} = -\pi/4$ and $\delta = \pi/2$, where θ_{23} is the mixing angle between mass eigenstates ν_2 and ν_3 , and δ is the CP violation phase in the standard parameterization of the mixing matrix [6]. The other mixing angles θ_{12} and θ_{13} are left free (independent on neutrino masses). The maximum value $\theta_{23} = -\pi/4$ is essential to explain the large atmospheric neutrino mixing. Also the prediction, $\delta = \pi/2$ is interesting under the situation where the search for the model

¹ In summary, we shall add a short remark for our results with the latest report on *Neutrino 2000* [3].

to predict the CP violation phase is an urgent problem. We examined the underlying symmetry for the democratic-type mass matrix and found that this mass matrix is derived by imposing the Z_3 symmetry for the model with two up-type doublet Higgs bosons [4, 5].

Here we want to consider the situation where the democratic-type mass matrix is derived from the dimension five effective interaction with one up-type doublet Higgs boson based on Z_3 symmetry. Let us define

$$\begin{aligned}\Psi_1 &= \frac{1}{\sqrt{3}}(\ell_e + \omega^2 \ell_\mu + \omega \ell_\tau) , \\ \Psi_2 &= \frac{1}{\sqrt{3}}(\ell_e + \omega \ell_\mu + \omega^2 \ell_\tau) , \\ \Psi_3 &= \frac{1}{\sqrt{3}}(\ell_e + \ell_\mu + \ell_\tau) ,\end{aligned}\tag{5}$$

where ℓ_i is the left-handed lepton doublet defined by, say, $\ell_e = (\nu_{eL}, e_L)^T$, and $\omega = \exp(i2\pi/3)$ which satisfies $\omega^3 = 1$ and $1 + \omega + \omega^2 = 0$. The fields Ψ_i behave as irreducible representations of Z_3 symmetry under the permutation of ℓ_e , ℓ_μ and ℓ_τ ,

$$\Psi_1 \rightarrow \omega \Psi_1 , \quad \Psi_2 \rightarrow \omega^2 \Psi_2 , \quad \Psi_3 \rightarrow \Psi_3 .\tag{6}$$

If we introduce a Higgs doublet that behaves $H_u \rightarrow \omega^2 H_u$, then we can construct the Z_3 invariant dimension five effective Lagrangian as

$$\mathcal{L}_0 = -(m_1^0 + \tilde{m}_1) \overline{(\Psi_1)^C} \Psi_1 \frac{H_u H_u}{u^2} - 2\tilde{m}_1 \overline{(\Psi_2)^C} \Psi_3 \frac{H_u H_u}{u^2} ,\tag{7}$$

where $u = \langle H_u \rangle$. Here we introduce two kinds of symmetry breaking terms, \mathcal{L}_1 and \mathcal{L}_2 as

$$\begin{aligned}\mathcal{L}_1 &= -(m_3^0 + \tilde{m}_3) \overline{(\Psi_3)^C} \Psi_3 \frac{H_u H_u}{u^2} - 2\tilde{m}_3 \overline{(\Psi_1)^C} \Psi_2 \frac{H_u H_u}{u^2} , \\ \mathcal{L}_2 &= -(m_2^0 + \tilde{m}_2) \overline{(\Psi_2)^C} \Psi_2 \frac{H_u H_u}{u^2} - 2\tilde{m}_2 \overline{(\Psi_1)^C} \Psi_3 \frac{H_u H_u}{u^2} .\end{aligned}\tag{8}$$

We remind that all types of Z_3 symmetry breaking terms are in \mathcal{L}_1 and \mathcal{L}_2 . Now we consider $\mathcal{L}_{\text{eff}} = \mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_2$. Then, after the symmetry breaking $\langle H_u \rangle = u$, we obtain the democratic-type mass matrix under the assumption that all coefficients m_i^0 and \tilde{m}_i are real (see Sec.2).

In this paper, we consider that the democratic-type mass matrix (or the dimension five effective interaction) is given at the right-handed neutrino mass scale, m_R and see the

predictions at m_Z by using the renormalization group. The renormalization group effect for the dimension five effective interaction have been examined intensively[7 - 15]. Casas *et al.* have investigated some general features of the quantum effects for the neutrino mixing matrix independent of the specific mass matrix [11]. Haba *et al.* have studied how the mixing angles behave by choosing the simple real mass matrix which explains the experimental data at m_Z [13].

We are interested in

- (i) whether our predicted value $\theta_{23} = -\pi/4$ is stable,
- (ii) how two angles θ_{12} and θ_{13} behave against quantum corrections, and
- (iii) whether the Dirac CP phase δ keeps maximal size at m_Z as that predicted at m_R .

First point (i) is indispensable for the democratic-type model to explain the atmospheric neutrino anomaly, and second point (ii) is interesting in view of searching the possibility that the small angle θ_{12} can be produced at m_Z from the large angle θ_{12} at m_R , or vice versa, in addition to their stable solutions. Third point (iii) is also important for future neutrino experiments in order to get the signals of CP and T violation in the lepton sector. These problems depend largely on the neutrino mass spectrum. In more restricted model with hierarchical mass spectrum, we investigated the points (i) and (ii), and showed that neutrino mixing angles are stable for the fully hierarchical mass spectrum, while the solar neutrino mixing angle is unstable for the hierarchical case with $m_1 \simeq m_2$, where m_1 and m_2 are the first and second mass eigenvalues [16]. For the general democratic-type model, in addition to the above results, we find the possibility that LMA and/or VO solutions can be realized at m_Z even if the solar neutrino mixing angle is small at m_R for the hierarchical case with $m_2/m_1 - 1 \ll 1$. Also in this case, Dirac CP phase is almost left maximal, and we can expect that CP and T violation effect can be detected in the future long baseline neutrino experiment such as neutrino factories [17].

This paper is organized as follows. In section 2, we briefly review the democratic-type neutrino mass matrix and its predictions. In section 3, we show the quantum corrections for the neutrino mass matrix. In section 4, we calculate the $\tan\beta$ dependence of neutrino masses, mixing angles and Dirac CP phase. In section 5, we give the results of numerical calculation and compare with the analytical estimation. Section 6 is devoted to the

summary.

2 Democratic-type mass matrix

Throughout of this paper, we assume that the mass matrix of charged leptons is diagonal. After symmetry breaking, we obtain the democratic-type neutrino mass matrix from Eqs.(7),(8) in the flavor eigenstate basis as[4, 5]

$$M_\nu(m_R) = \sum_{j=1}^3 (m_j^0 S_j + \tilde{m}_j T_j) , \quad (9)$$

where

$$\begin{aligned} S_1 &= \frac{1}{3} \begin{pmatrix} 1 & \omega^2 & \omega \\ \omega^2 & \omega & 1 \\ \omega & 1 & \omega^2 \end{pmatrix} , \quad S_2 = \frac{1}{3} \begin{pmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{pmatrix} , \quad S_3 = \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} , \\ T_1 &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix} , \quad T_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix} , \quad T_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} . \end{aligned} \quad (10)$$

Six quantities m_j^0 and \tilde{m}_j ($j = 1, 2, 3$) called mass parameters are taken to be real, and the form in Eq.(9) is assumed to be generated from the effective dimension-five operators at m_R .

$M_\nu(m_R)$ can be transformed into real symmetric matrix $\overline{M}_\nu(m_R) = V_{\text{Tri}}^T M_\nu(m_R) V_{\text{Tri}}$, where V_{Tri} is the following tri-maximal mixing matrix as

$$V_{\text{Tri}} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ \omega & \omega^2 & 1 \\ \omega^2 & \omega & 1 \end{pmatrix} . \quad (11)$$

Then, the neutrino mixing matrix U (MNS matrix [18]) which diagonalizes $M_\nu(m_R)$ as $U^T M_\nu(m_R) U = D_\nu \equiv \text{diag}(m_1, m_2, m_3)$ is expressed as $U = V_{\text{Tri}} O$, where O is the orthogonal matrix which diagonalizes $\overline{M}_\nu(m_R)$, and m_i ($i = 1, 2, 3$) are neutrino mass eigenvalues. This expression leads to the condition that the mixing matrix U should satisfy $U_{\mu i} = U_{\tau i}^*$ ($i = 1, 2, 3$), which do not depend on real mass parameters. From these conditions on the mixing matrix, we find that $c_{23}^2 = s_{23}^2 = 1/2$ and $\cos \delta = 0$ by using

the standard parameterization advocated in [6]. Here $c_{ij} = \cos \theta_{ij}$, $s_{ij} = \sin \theta_{ij}$ with the mixing angle θ_{ij} between mass eigenstates ν_i and ν_j , and δ is the Dirac CP phase. Thus, we can obtain the following expression of U as

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\rho} & 0 \\ 0 & 0 & e^{-i\rho} \end{pmatrix} \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & -is_{13} \\ -\frac{s_{12}-ic_{12}s_{13}}{\sqrt{2}} & \frac{c_{12}+is_{12}s_{13}}{\sqrt{2}} & -\frac{c_{13}}{\sqrt{2}} \\ -\frac{s_{12}+ic_{12}s_{13}}{\sqrt{2}} & \frac{c_{12}-is_{12}s_{13}}{\sqrt{2}} & \frac{c_{13}}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & i \end{pmatrix}, \quad (12)$$

where we have taken $s_{23} = -c_{23} = -1/\sqrt{2}$ and $\delta = \pi/2$. The quantity ρ is a redundant phase which can be absorbed into charged leptons by the phase redefinition. $\text{Diag}(1, 1, i)$ is the Majorana phase matrix, which shows no CP violation intrinsic to Majorana neutrinos. Indeed, the phase i relates to CP signs of neutrino masses in addition to their relative sign assignments. In this model, therefore, six real mass parameters (m_i^0, \tilde{m}_i ($i = 1, 2, 3$)) are changed into three neutrino masses (m_1, m_2, m_3), two mixing angles (θ_{12}, θ_{13}), and one unphysical phase (ρ).

Since three neutrino mass eigenvalues are free parameters, we adopt the following mass squared differences as

$$\Delta m_{\text{atm}}^2 \equiv |\Delta m_{32}^2| \sim |\Delta m_{31}^2|, \quad |\Delta m_{12}^2| \ll \Delta m_{\text{atm}}^2, \quad (13)$$

where $\Delta m_{ij}^2 \equiv m_i^2 - m_j^2$. Under the assignment of Eq.(13), we consider the following mass spectrum as

$$\begin{aligned} \text{Hierarchical case} & : m_1 \simeq m_2 \ll m_3, \\ \text{Inversely hierarchical case} & : m_1 \simeq m_2 \gg m_3, \end{aligned} \quad (14)$$

and we assume all mass eigenvalues are positive². Of course, there are another mass spectrums which satisfies Eq.(13); fully hierarchical case ($m_1 \ll m_2 \ll m_3$) and nearly degenerate case ($m_1 \simeq m_2 \simeq m_3$) [19]. However, for the former case, quantum corrections hardly change the structure of the neutrino mass matrix, and hence all physical quantities are stable. For the latter case, neutrino mixing is highly sensitive to the input values at

² In this spectrum, the behavior of the mixing matrix hardly depends on the sign of m_3 . So we can generally take $m_2 > 0$. Then, the behavior of the mixing matrix depends on the relative sign of m_1 and m_2 as well as their absolute sizes. For $m_1 < 0$, mixing angles are stable while CP violation phase is unstable. See Ref. [16].

m_R , and it is laborious to obtain their analytical expressions. It is also noted that we do not adopt that $|\Delta m_{12}^2|$ is the mass squared difference for the solar neutrino mixing. This is because mass eigenvalues m_i ($i = 1, 2, 3$) are those given at m_R , and as we shall see later, $|\Delta m_{12}^2|$ varies while $|\Delta m_{32}^2|$ is almost stable against quantum corrections, when the mass spectrum is given as Eq.(14).

3 Quantum corrections

The neutrino mass matrix, and hence the neutrino mixing matrix, may vary by quantum corrections [7, 8]. In the minimal supersymmetric standard model (MSSM) with dimension-five operators which give Majorana masses to left-handed neutrinos, quantum corrections appear in the relation of the Majorana mass matrices between at m_R and at m_Z as [13]

$$M_\nu(m_Z) = \begin{pmatrix} \frac{1}{\sqrt{I_e}} & 0 & 0 \\ 0 & \frac{1}{\sqrt{I_\mu}} & 0 \\ 0 & 0 & \frac{1}{\sqrt{I_\tau}} \end{pmatrix} M_\nu(m_R) \begin{pmatrix} \frac{1}{\sqrt{I_e}} & 0 & 0 \\ 0 & \frac{1}{\sqrt{I_\mu}} & 0 \\ 0 & 0 & \frac{1}{\sqrt{I_\tau}} \end{pmatrix}, \quad (15)$$

where I_i ($i = e, \mu, \tau$) are defined as

$$I_i = \exp \left(\frac{1}{8\pi^2} \int_{\ln(m_R)}^{\ln(m_Z)} y_i^2 dt \right). \quad (16)$$

Here y_i are Yukawa couplings of charged leptons in the mass eigenstate, $t = \ln \mu$ with the renormalization point μ , and overall renormalization effect has been absorbed into $M_\nu(m_R)$.

According to the discussion in [13], the approximation of $\sqrt{I_j/I_\tau} \sim 1/\sqrt{I_\tau}$ ($j = e, \mu$) is held with good accuracy in the region of $2 < \tan \beta < 60$. Here $\tan \beta = \langle \phi_u \rangle / \langle \phi_d \rangle$, in which ϕ_u and ϕ_d are two Higgs doublets in the MSSM. By using this approximation, $M_\nu(m_Z)$ reduces to the following simpler form as

$$M_\nu(m_Z) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{\sqrt{I_\tau}} \end{pmatrix} M_\nu(m_R) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{\sqrt{I_\tau}} \end{pmatrix} = M_\nu(m_R) - \epsilon M_1 + O(\epsilon^2), \quad (17)$$

where

$$M_1 = \begin{pmatrix} 0 & 0 & (M_\nu(m_R))_{13} \\ 0 & 0 & (M_\nu(m_R))_{23} \\ (M_\nu(m_R))_{13} & (M_\nu(m_R))_{23} & 2(M_\nu(m_R))_{33} \end{pmatrix}, \quad (18)$$

and ϵ is defined as

$$\epsilon = 1 - \frac{1}{\sqrt{I_\tau}} = 1 - \left(\frac{m_Z}{m_R} \right)^{\frac{1}{8\pi^2}(1+\tan^2\beta)(m_\tau/v)^2} > 0, \quad (19)$$

with the mass of τ lepton, m_τ , and $v^2 = \langle \phi_u \rangle^2 + \langle \phi_d \rangle^2$. In the second equality in Eq.(19), we have neglected the running effect of y_τ . In order to estimate the value of ϵ , we consider the right-handed neutrino mass scale m_R as 10^{13} GeV. Then, with $m_Z = 91.187$ GeV, $m_\tau = 1.777$ GeV and $v = 245.4$ GeV, we find

$$8 \times 10^{-5} < \epsilon < 6 \times 10^{-2} \quad (\text{for } 2 < \tan\beta < 60). \quad (20)$$

Therefore, we neglect the $O(\epsilon^2)$ terms in Eq.(17) when obtaining the analytical expression of the mixing angles and neutrino masses at m_Z .

By using Eq.(17), we can obtain the Majorana neutrino mass matrix at m_Z . This mass matrix depends on $\tan\beta$ via ϵ as well as neutrino masses, mixing angles and CP phases given at m_R . We take θ_{23} and CP phases at m_R as those predicted by the democratic-type model. We also assume that neutrino masses at m_R is given as Eqs.(13) and (14). Then we can investigate the $\tan\beta$ dependence of the neutrino mixing matrix at m_Z , which includes two mixing angles, θ_{12} , θ_{13} , and neutrino masses at m_R . As we will show in the next section, θ_{13} hardly depends on quantum corrections, so we can take the most stringent constraint on θ_{13} , $s_{13} = 0.16$, from the CHOOZ data [20].

We also check the analytical estimation by numerical calculation, which will be shown in section 5. For any $\tan\beta$, we calculate the mass matrix $M_\nu(m_Z)$ numerically by using Mathematica, and find the unitary matrix \hat{U} which diagonalizes $M_\nu(m_Z)$ as $\hat{U}^T M_\nu(m_Z) \hat{U}$. Hereafter, we denote physical quantities at m_Z as $\hat{\theta}$, \hat{m}_1 and so on. $\tan\beta$ dependence of the mixing angle $\hat{\theta}$ are shown by using the following expression as

$$\begin{aligned} \sin^2 2\hat{\theta}_{13} &= 4|\hat{U}_{e3}|^2(1 - |\hat{U}_{e3}|^2), \\ \sin^2 2\hat{\theta}_{23} &= 4 \frac{|\hat{U}_{\mu 3}|^2}{1 - |\hat{U}_{e3}|^2} \left(1 - \frac{|\hat{U}_{\mu 3}|^2}{1 - |\hat{U}_{e3}|^2} \right), \end{aligned}$$

$$\sin^2 2\hat{\theta}_{12} = 4 \frac{|\hat{U}_{e2}|^2}{1 - |\hat{U}_{e3}|^2} \left(1 - \frac{|\hat{U}_{e2}|^2}{1 - |\hat{U}_{e3}|^2} \right). \quad (21)$$

4 Quantum effects for physical quantities

In this section, we show the $\tan \beta$ dependence of neutrino masses, mixing angles and Dirac CP phase at m_Z .

4.1 neutrino masses and mixing angles

By transforming $M_\nu(m_Z)$ by U in Eq.(12), we obtain the mass matrix $\widetilde{M}_\nu(m_Z)$ by keeping ϵ up to the first order as

$$\begin{aligned} \widetilde{M}_\nu(m_Z) &= U^T M_\nu(m_Z) U \\ &= \begin{pmatrix} (1 - \epsilon|p|^2)m_1 & \frac{1}{2}\epsilon(m_1 p q^* + m_2 p^* q) & i\epsilon P \\ \frac{1}{2}\epsilon(m_1 p q^* + m_2 p^* q) & (1 - \epsilon|q|^2)m_2 & -i\epsilon Q \\ i\epsilon P & -i\epsilon Q & (1 - \epsilon c_{13}^2)m_3 \end{pmatrix}, \end{aligned} \quad (22)$$

where

$$\begin{aligned} p &\equiv s_{12} - i c_{12} s_{13}, \quad q \equiv c_{12} + i s_{12} s_{13}, \\ P &\equiv \frac{1}{2} c_{13} (m_1 p - m_3 p^*), \quad Q \equiv \frac{1}{2} c_{13} (m_2 q - m_3 q^*). \end{aligned} \quad (23)$$

Let us define the submatrices as

$$\begin{aligned} \boldsymbol{\mu} &= \begin{pmatrix} (1 - \epsilon|p|^2)m_1 & \frac{1}{2}\epsilon(m_1 p q^* + m_2 p^* q) \\ \frac{1}{2}\epsilon(m_1 p q^* + m_2 p^* q) & (1 - \epsilon|q|^2)m_2 \end{pmatrix}, \quad \boldsymbol{m} = \begin{pmatrix} i\epsilon P \\ -i\epsilon Q \end{pmatrix}, \\ M &= (1 - \epsilon c_{13}^2)m_3. \end{aligned} \quad (24)$$

Then, \boldsymbol{m} is much smaller than either M in the hierarchical case or $\boldsymbol{\mu}$ in the inversely hierarchical case. So we can block diagonalize \widetilde{M}_ν by using seesaw expansion in [21] as

$$U_{\text{seesaw}}^T \widetilde{M}_\nu U_{\text{seesaw}} \simeq \begin{pmatrix} \boldsymbol{\mu} & \mathbf{0} \\ \mathbf{0} & M \end{pmatrix}, \quad (25)$$

where

$$U_{\text{seesaw}} \simeq \begin{pmatrix} \mathbf{1}_2 & iS \\ iS^\dagger & 1 \end{pmatrix}, \quad (26)$$

with a 2 by 2 unit matrix $\mathbf{1}_2$, and

$$iS \simeq \begin{cases} (M^{-1}\mathbf{m}^T)^\dagger \simeq M^{-1} \begin{pmatrix} -i\epsilon P^* \\ i\epsilon Q^* \end{pmatrix} & \text{(hierarchical case) ,} \\ -\boldsymbol{\mu}^{-1}\mathbf{m} \simeq \mu^{-1} \begin{pmatrix} -i\epsilon P m_2 \\ i\epsilon Q m_1 \end{pmatrix} & \text{(inversely hierarchical case) ,} \end{cases} \quad (27)$$

where $\mu \equiv \det \boldsymbol{\mu}$. Here we have neglected the normalization factor of U_{seesaw} since it is nearly unity. By keeping ϵ up to the first order, U_{seesaw} is simply rewritten as

$$U_{\text{seesaw}} \simeq \begin{pmatrix} 1 & 0 & \pm \frac{i}{2}\epsilon c_{13}p \\ 0 & 1 & \mp \frac{i}{2}\epsilon c_{13}q \\ \pm \frac{i}{2}\epsilon c_{13}p^* & \mp \frac{i}{2}\epsilon c_{13}q^* & 1 \end{pmatrix}. \quad (28)$$

Here, the upper (lower) sign is for the hierarchical (inversely hierarchical) case, where we have neglected $O(m_{1,2}/m_3)$ ($O(m_3/m_{1,2})$) terms .

Now, in order to obtain the neutrino mixing matrix \hat{U} , we only have to diagonalize the submatrix $\boldsymbol{\mu}$ in Eq.(25). Let us define the small parameter

$$\xi = 1 - \frac{m_2}{m_1}. \quad (29)$$

By keeping ϵ and ξ up to the first order, $\boldsymbol{\mu}$ is rewritten as

$$\boldsymbol{\mu} \simeq \begin{pmatrix} 1 - \epsilon|p|^2 & \frac{1}{2}\epsilon \sin 2\theta_{12}c_{13}^2 \\ \frac{1}{2}\epsilon \sin 2\theta_{12}c_{13}^2 & 1 - \epsilon|q|^2 - \xi \end{pmatrix} m_1. \quad (30)$$

It is easy to diagonalize $\boldsymbol{\mu}$ since it is a 2 by 2 real symmetric matrix. By diagonalizing $\boldsymbol{\mu}$ as $\tilde{U}_{12}^T \boldsymbol{\mu} \tilde{U}_{12}$ in which

$$\tilde{U}_{12} = \begin{pmatrix} \cos \tilde{\theta} & \sin \tilde{\theta} \\ -\sin \tilde{\theta} & \cos \tilde{\theta} \end{pmatrix} = \begin{pmatrix} \tilde{c} & \tilde{s} \\ -\tilde{s} & \tilde{c} \end{pmatrix}, \quad (31)$$

we get

$$\tan 2\tilde{\theta} \simeq -\frac{\epsilon \sin 2\theta_{12}c_{13}^2}{\epsilon \cos 2\theta_{12}c_{13}^2 + \xi}. \quad (32)$$

Then, the neutrino mixing matrix \hat{U} is given as

$$\hat{U} \simeq U U_{\text{seesaw}} \tilde{U}, \quad (33)$$

and each element is written as follows

$$\begin{aligned}
\hat{U}_{e1} &\simeq c_{13} \left[c'_{12} \left(1 \mp \frac{1}{2} \epsilon s_{13}^2 \right) \pm \frac{i}{2} \epsilon s'_{12} s_{13} \right] , \\
\hat{U}_{e2} &\simeq c_{13} \left[s'_{12} \left(1 \mp \frac{1}{2} \epsilon s_{13}^2 \right) \mp \frac{i}{2} \epsilon c'_{12} s_{13} \right] , \\
\hat{U}_{e3} &\simeq s_{13} \left(1 \pm \frac{1}{2} \epsilon c_{13}^2 \right) , \\
\hat{U}_{\mu 1} &\simeq -\frac{1}{\sqrt{2}} \left[s'_{12} \left(1 \mp \frac{1}{2} \epsilon c_{13}^2 \right) - i c'_{12} s_{13} \left(1 \pm \frac{1}{2} \epsilon c_{13}^2 \right) \right] , \\
\hat{U}_{\mu 2} &\simeq \frac{1}{\sqrt{2}} \left[c'_{12} \left(1 \mp \frac{1}{2} \epsilon c_{13}^2 \right) + i s'_{12} s_{13} \left(1 \pm \frac{1}{2} \epsilon c_{13}^2 \right) \right] , \\
\hat{U}_{\mu 3} &\simeq -\frac{i}{\sqrt{2}} c_{13} \left(1 \pm \frac{1}{2} \epsilon c_{13}^2 \right) , \\
\hat{U}_{\tau 1} &\simeq -\frac{1}{\sqrt{2}} (s'_{12} + i c'_{12} s_{13}) \left(1 \pm \frac{1}{2} \epsilon c_{13}^2 \right) , \\
\hat{U}_{\tau 2} &\simeq \frac{1}{\sqrt{2}} (c'_{12} - i s'_{12} s_{13}) \left(1 \pm \frac{1}{2} \epsilon c_{13}^2 \right) , \\
\hat{U}_{\tau 3} &\simeq \frac{i}{\sqrt{2}} \left[1 \pm \frac{1}{2} \epsilon (1 + s_{13}^2) \right] ,
\end{aligned} \tag{34}$$

where

$$c'_{12} \equiv \cos \theta'_{12} = \cos (\theta_{12} + \tilde{\theta}) , \quad s'_{12} \equiv \sin \theta'_{12} = \sin (\theta_{12} + \tilde{\theta}) , \tag{35}$$

and we have neglected an unphysical phase.

Thus, we obtain the mixing angle at m_Z as

$$\begin{aligned}
\sin^2 2\hat{\theta}_{13} &\simeq \sin^2 2\theta_{13} (1 \pm \epsilon \cos 2\theta_{13}) , \\
\sin^2 2\hat{\theta}_{23} &\simeq 1 + O(\epsilon^2) , \\
\sin^2 2\hat{\theta}_{12} &\simeq \sin^2 2\theta'_{12} (1 + O(\epsilon^2)) \\
&= \frac{(\xi \sin 2\theta_{12})^2}{(\epsilon c_{13}^2 + \xi \cos 2\theta_{12})^2 + (\xi \sin 2\theta_{12})^2} (1 + O(\epsilon^2)) .
\end{aligned} \tag{36}$$

Therefore, we may say that there are no extra mixings between $\nu_{1,2}$ and ν_3 by the renormalization group equation, where ν_i is the mass eigenstate at m_R . That is, mixing angles θ_{13} and $\theta_{23} = -\pi/4$ are essentially stable against quantum corrections. Also the mixing angle $\hat{\theta}_{12}$ is almost equal to $\theta'_{12} = \theta_{12} + \tilde{\theta}$, and hence, we may roughly obtain \hat{U} by changing from θ_{12} to θ'_{12} in U . In other words, the behavior of the mixing angles hardly depends on the contribution from seesaw expansion.

Next, we estimate the neutrino mass eigenvalues \hat{m}_i ($i = 1, 2, 3$) at m_Z . They are given as

$$\begin{aligned}\hat{m}_1 &\simeq \frac{m_1}{2} \left[2 - \epsilon(1 + s_{13}^2) - \xi + \text{sign}(\xi) \sqrt{(\xi + \epsilon \cos 2\theta_{12} c_{13}^2)^2 + (\epsilon \sin 2\theta_{12} c_{13}^2)^2} \right], \\ \hat{m}_2 &\simeq \frac{m_1}{2} \left[2 - \epsilon(1 + s_{13}^2) - \xi - \text{sign}(\xi) \sqrt{(\xi + \epsilon \cos 2\theta_{12} c_{13}^2)^2 + (\epsilon \sin 2\theta_{12} c_{13}^2)^2} \right], \\ \hat{m}_3 &\simeq (1 - \epsilon c_{13}^2) m_3,\end{aligned}\tag{37}$$

where $\text{sign}(\xi)$ is $+1(-1)$ for $\xi > 0(\xi < 0)$. Then, the mass squared difference $|\Delta \hat{m}_{12}^2|$ is given as

$$|\Delta \hat{m}_{12}^2| \simeq 2m_1^2 \sqrt{(\xi + \epsilon \cos 2\theta_{12} c_{13}^2)^2 + (\epsilon \sin 2\theta_{12} c_{13}^2)^2}.\tag{38}$$

As one can understand by looking at $\sin^2 2\hat{\theta}_{12}$ in Eq.(36) and at Eq.(38), $\tan \beta$ dependences of $\sin^2 2\hat{\theta}_{12}$ and $|\Delta \hat{m}_{12}^2|$ depend on the sign of ξ . Therefore, we consider the following two cases :

Case (a) : $\xi > 0$ ($m_1 > m_2$)

In this case, the first term of the denominator of $\sin^2 2\hat{\theta}_{12}$ increases monotonously as $\tan \beta$ grows³. Thus, $\sin^2 2\hat{\theta}_{12}$ becomes smaller as the quantum corrections become larger. Hence, one may expect that SMA solution at m_Z is realized from the large mixing angle at m_R . However, such undertaking is in vain. Let us explain the reason briefly. If one would try to find the parameter regions of Δm_{12}^2 and ϵ to produce SMA solution, one should at least set $|\Delta \hat{m}_{12}^2| \simeq \Delta m_{\text{SMA}}^2$ and $\sin^2 2\hat{\theta}_{12} \simeq \sin^2 2\theta_{\text{SMA}}$. By simplifying $c_{13}^2 = 1$ from the CHOOZ bound, one could obtain

$$\begin{aligned}|\Delta m_{12}^2| &\simeq \Delta m_{\text{SMA}}^2 \left| \frac{\sin 2\theta_{\text{SMA}}}{\sin 2\theta_{12}} \right|, \\ \epsilon &\simeq \frac{\Delta m_{\text{SMA}}^2}{2m_1^2} |\cos 2\theta_{\text{SMA}}| (1 - \cot 2\theta_{12} \tan 2\theta_{\text{SMA}}).\end{aligned}\tag{39}$$

The first relation in Eq.(39) shows that $|\Delta m_{12}^2|$ is needed to be about 1/10 times as small as Δm_{SMA}^2 when θ_{12} is large at m_R . For example, we can obtain $|\Delta m_{12}^2| \simeq 10^{-7} \sim 10^{-6} \text{ eV}^2$

³ Here we set $0 \leq \theta_{12} \leq \pi/4$ since we consider that electron neutrinos mainly consist of ν_1 if the solar neutrino mixing is small at m_R . When $\theta_{12} = \pi/4$, i.e., solar mixing angle is exactly maximal at the m_R scale, the dependences of $\sin^2 2\hat{\theta}_{12}$ and $\Delta \hat{m}_{12}^2$ do not depend on the sign of ξ .

when $\sin 2\theta_{12} = \sqrt{8/9}$ ⁴. In this case, however, the quantity Δm_{12}^2 is negative at m_Z , and as a result the matter effect cannot take place.

Case (b) : $\xi < 0$ ($m_1 < m_2$)

In this case, the first term of the denominator of $\sin^2 2\hat{\theta}_{12}$ in Eq.(36) can become 0 when

$$\epsilon \simeq |\xi| \cos 2\theta_{12} = \frac{|\Delta m_{12}^2|}{2m_1^2} \cos 2\theta_{12} , \quad (40)$$

and at the same time $\sin^2 2\hat{\theta}_{12}$ becomes maximal. Here we have used $c_{13}^2 = 1$ for simplicity. Hence we can expect that the large mixing angle such as LMA and VO solutions at m_Z can be realized even from the small mixing angle at m_R . By substituting Eq.(40) into Eq.(38), we obtain the initial mass splitting as

$$|\Delta m_{12}^2| \simeq \frac{\Delta \hat{m}_{12}^2}{\sin 2\theta_{12}} . \quad (41)$$

Thus, the mass splitting at m_R is about 10 times as large as that at m_Z when θ_{12} is the small mixing angle. By setting $\sin 2\theta_{12} = 0.1$ such as preferred by SMA solution, for example, we obtain $|\Delta m_{12}^2| \simeq 1 \times 10^{-4} \sim 1 \times 10^{-3} \text{ eV}^2$ for LMA and $|\Delta m_{12}^2| \simeq 10^{-10} \sim 10^{-9} \text{ eV}^2$ for VO solutions.

In the inversely hierarchical case with $\Delta m_{\text{atm}}^2 = 3.5 \times 10^{-3} \text{ eV}^2$ and $s_{13} = 0.16$, we obtain $\epsilon \simeq 0.01 \sim 0.1$ for LMA, and $\epsilon \simeq 10^{-8} \sim 10^{-7}$ for VO solutions, from Eq.(40). Thus, LMA solution can be generated for $\tan \beta \gtrsim 30$, while $\tan \beta$ is too tiny to produce VO solution for the realistic $\tan \beta$ region.

On the contrary, in the hierarchical case, m_1^2 is restricted to about $2 \times 10^{-3} \text{ eV}^2$ for LMA solution since $|\Delta m_{12}^2|$ can become as large as $1 \times 10^{-3} \text{ eV}^2$, which leads to the results that LMA solution can be realized for the large $\tan \beta$ region, i.e., $\tan \beta \gtrsim 40$. However, we may say $m_1^2 \simeq 10^{-8} \sim 10^{-3} \text{ eV}^2$ for VO solution, and VO solution can be realized for wide $\tan \beta$ region.

⁴This angle has been predicted in a restricted democratic-type model [5].

4.2 CP violation phase

Now, let us take our attention to the CP phase. From Eq.(34), we get the Jarlskog parameter \hat{J} as

$$|\hat{J}| = |\text{Im}(\hat{U}_{e1}\hat{U}_{\mu 2}\hat{U}_{e2}^*\hat{U}_{\mu 1}^*)| \simeq \frac{1}{4}s_{13}c_{13}^2 \sin 2\theta'_{12} \left[1 \pm \frac{1}{2}\epsilon(1 - 3s_{13}^2) \right]. \quad (42)$$

The deviation from unity in the bracket means the contribution from seesaw expansion. This shows that the $\tan\beta$ dependence of the Jarlskog parameter is almost same as that of $\sin 2\theta'_{12}$. That is, \hat{J} is damping as $\tan\beta$ grows for $\xi > 0$, while it has a peak for $\xi < 0$.

Dirac CP phase $\hat{\delta}$ is also given as

$$\begin{aligned} |\sin \hat{\delta}| &= \frac{|\text{Im}(\hat{U}_{e1}\hat{U}_{\mu 2}\hat{U}_{e2}^*\hat{U}_{\mu 1}^*)|}{|\hat{U}_{e1}||\hat{U}_{e2}||\hat{U}_{e3}||\hat{U}_{\mu 3}||\hat{U}_{\tau 3}|}(1 - |\hat{U}_{e3}|^2) \\ &\simeq \sqrt{\frac{\sin^2 2\theta'_{12}}{\sin^2 2\theta'_{12} + (\epsilon s_{13})^2}} \\ &\simeq \sqrt{\frac{(\xi \sin 2\theta_{12})^2}{(\xi \sin 2\theta_{12})^2 [1 + (\epsilon s_{13})^2] + (\epsilon s_{13})^2(\epsilon c_{13}^2 + \xi \cos 2\theta_{12})^2}}. \end{aligned} \quad (43)$$

Without the contribution from seesaw expansion, we could not look at the corrections of the denominator of the middle in Eq.(43), $(\epsilon s_{13})^2$, and $|\sin \hat{\delta}|$ would not depend on the quantum correction. In other words, we may say that Dirac CP phase is stable for the small $\tan\beta$ region whatever ξ and θ_{12} are given at m_R . However, $(\epsilon s_{13})^2$ have a possibility of becoming comparable to the $\sin^2 2\theta'_{12}$ in the large $\tan\beta$ region, and we should take this contribution into account to compare with the numerical evaluation.

5 Numerical check

We calculated the numerical evaluation of neutrino masses and the mixing matrix at m_Z to compare with the analytical estimation shown in the previous section. To simplify the analysis, we evaluated in the inversely hierarchical case, where the mass spectrum at m_R is easy to be determined except for $|\Delta m_{12}^2|$. Figs. 1 and 2 are the results of the $\tan\beta$ dependence of mixing angles, Dirac CP phase and mass squared differences at m_Z .

(1) An example of the case (a)

In this case, $\sin^2 2\hat{\theta}_{12}$ at m_Z becomes small for $\tan \beta > 4$ as we can see from Eq.(36). The numerical analysis is performed to see the $\tan \beta$ dependence for $\sin^2 2\hat{\theta}_{12}$ and other quantities in detail and the result is shown in Fig. 1. As we can see, $\sin^2 2\hat{\theta}_{12}$ can become as small as 0.01 at around $\tan \beta \sim 8$, though $\sin^2 2\theta_{12} = 8/9$ at m_R . The mass squared difference $|\Delta m_{12}^2|$ at m_Z become large. For larger values of $\tan \beta$, $|\sin \hat{\delta}|$ becomes small, though $|\sin \delta| = 1$ at m_R . Other quantities, $\hat{\theta}_{13}$, $\hat{\theta}_{23}$ and $|\Delta \hat{m}_{23}^2|$ do not change much.

From Fig.1, one might think that this instability can be used to realize SMA solution. Unfortunately, this is not true, because $(m_2^2 - m_1^2) \cos 2\hat{\theta}_{12} < 0$ at m_Z so that the MSW mechanism does not work.

(2) An example of the case (b)

In Fig. 2, we take as input values $m_1 = \sqrt{\Delta m_{\text{atm}}^2}$, $m_2 = \sqrt{\Delta m_{\text{atm}}^2 + |\Delta m_{12}^2|}$, $m_3 = 0$, $\Delta m_{\text{atm}}^2 = 3.4 \times 10^{-3}$ (eV²), $|\Delta m_{12}^2| = 1 \times 10^{-4}$ (eV²), $\sin 2\theta_{12} = 0.1$, $s_{13} = 0.16$ with the predicted values of $\theta_{23} = -\pi/4$ and $\delta = \pi/2$ at m_R . $\hat{\theta}_{13}$, $\hat{\theta}_{23}$ and $|\Delta \hat{m}_{23}^2|$ are hardly dependent on the quantum corrections similarly as in the case (a). $\sin^2 2\hat{\theta}_{12}$ has a peak around $\tan \beta \sim 30$. From Eq.(40), we obtain the value of ϵ at the peak as 1.5×10^{-2} , which corresponds to $\tan \beta \sim 30$, which is consistent with the figure. $|\Delta \hat{m}_{12}^2|$ is decreasing till $\tan \beta \sim 30$ and then increasing, which is caused by the negative ξ in Eq.(38). $|\sin \hat{\delta}|$ is almost maximal in the wide $\tan \beta$ region, which is due to larger Δm_{12}^2 than that in the previous example, and the effect of quantum corrections disappears. Thus, the selection of those parameter values above gives an example of generating LMA solution with maximal CP phase at m_Z .

6 Summary

We investigated the quantum effects for the democratic-type neutrino mass matrix with the (inversely) hierarchical mass spectrum. We assumed that this mass matrix is generated by dimension-five operators added in the MSSM at m_R , and considered the mass matrix at m_Z by using the renormalization group. We summarize our results as follows :

- $\theta_{23} = -\pi/4$, θ_{13} and $|\Delta m_{23}^2|$ are almost stable against quantum corrections.

- Dirac CP phase $\delta = \pi/2$ is almost stable unless the input mass splitting $\xi = (m_1 - m_2)/m_1$ at m_R is too small.
- The behavior of the mixing angle θ_{12} and Δm_{12}^2 is divided into two cases :

- Case (a) : $m_1 > m_2$ case

In this case $\sin^2 2\theta_{12}$ is damping dependent on the size of the mass difference ξ . From this nature, we expected the possibility of obtaining SMA solution at m_Z even if the solar neutrino mixing angle is large at m_R . Though the mixing angle can become as small as the region preferred by SMA solution, $\Delta m_{12}^2 \cos 2\hat{\theta}_{12}$ at m_Z is negative and the matter effect cannot occur in this case.

- Case (b) : $m_1 < m_2$ case

In this case $\sin^2 2\theta_{12}$ has a peak which is dependent on the size of ξ . From this nature, we showed the possibility of obtaining LMA and/or VO solutions at m_Z even if the solar neutrino mixing angle is small at m_R . For the inversely hierarchical case, we can obtain LMA solution in the region of $\tan \beta \gtrsim 30$. For the hierarchical case, we can obtain either LMA solution in the region of $\tan \beta \gtrsim 40$ or VO solution in the wide $\tan \beta$ range. In order for this phenomena to occur, Δm_{12}^2 at m_R should be taken about 10 times as large as the experimental data.

From the above results, our democratic-type model can give the nearly maximal mixing angle for the atmospheric neutrino anomaly at m_Z , provided the free parameter θ_{13} is taken to be as small as that preferred by the CHOOZ data.

As to the solar neutrino problem, the latest report from Super-Kamiokande shows that SMA and VO solutions are disfavored by comparing the day/night spectrum with the results of the flux global analysis [3]. If we consider this new data seriously, putting too strong degeneracy on m_1 and m_2 , i.e., $\xi \ll 1$, at m_R is limited in the small $\tan \beta$ region, otherwise the solar neutrino mixing angle would become small at m_Z no matter how large it could be taken at m_R .

The result that $\delta = \pi/2$ predicted by the democratic-type model is almost kept maximal is the best situation to search the CP and T violation phenomenon in the near future

projects like neutrino factories. Thus, if the solar neutrino problem will be solved by LMA solution, our model will be checked by looking at the signals of CP and T violation by the neutrino oscillation experiments in the next century.

Acknowledgment

This work is supported in part by the Japanese Grant-in-Aid for Scientific Research of Ministry of Education, Science, Sports and Culture, No.12047218.

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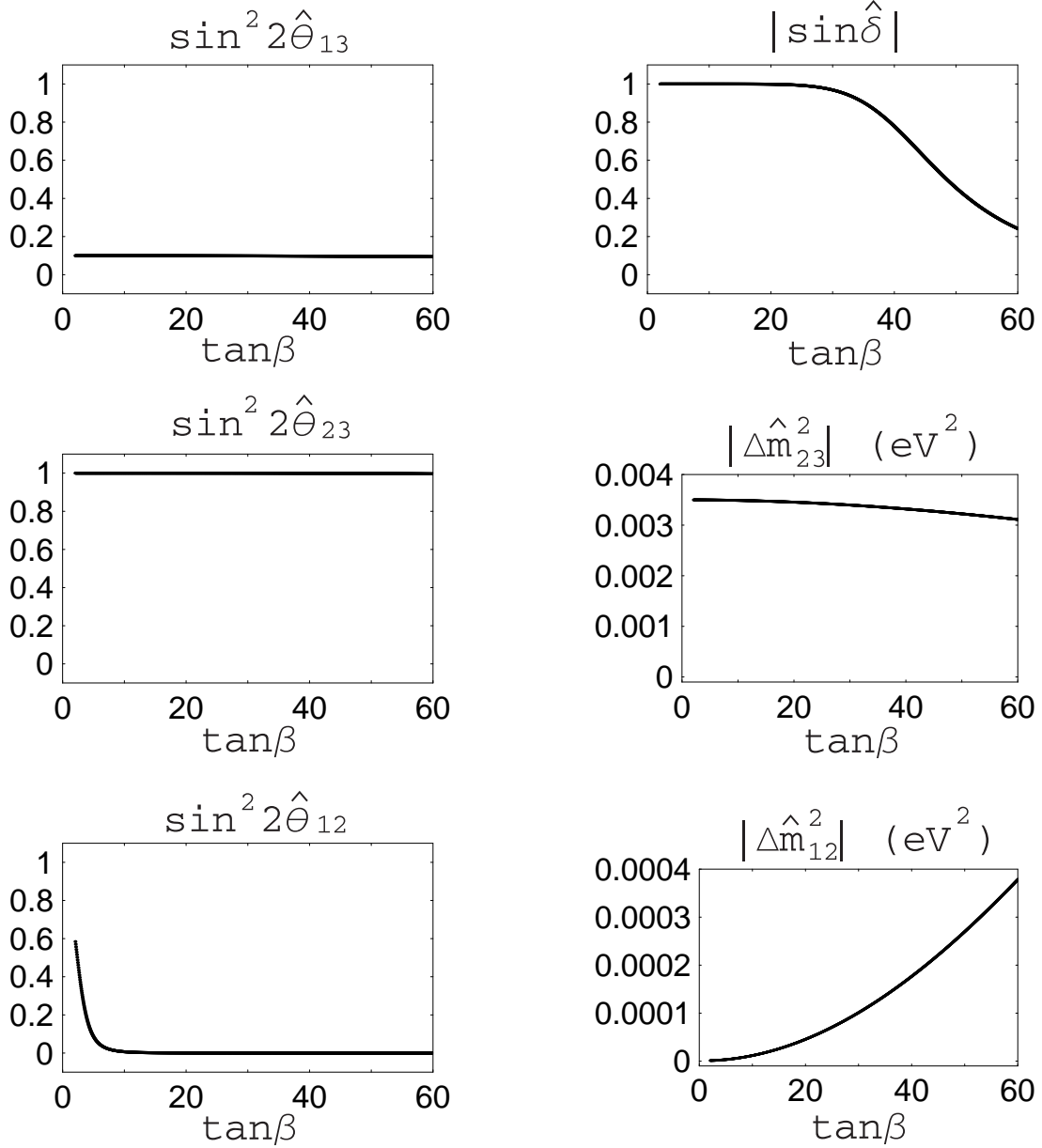


Figure 1: $\tan\beta$ dependence of neutrino mixing angles, Dirac CP phase and mass squared differences for $m_1 > m_2 \gg m_3$ at m_R . As initial values at m_R , we took $(m_1, m_2, m_3) = (5.9169 \times 10^{-2}, 5.9161 \times 10^{-2}, 0)\text{eV}$, $\sin 2\theta_{12} = \sqrt{8/9}$, $s_{13} = 0.16$. We reminded that $\theta_{23} = -\pi/4$ and $\delta = \pi/2$, which are the predictions of our model.

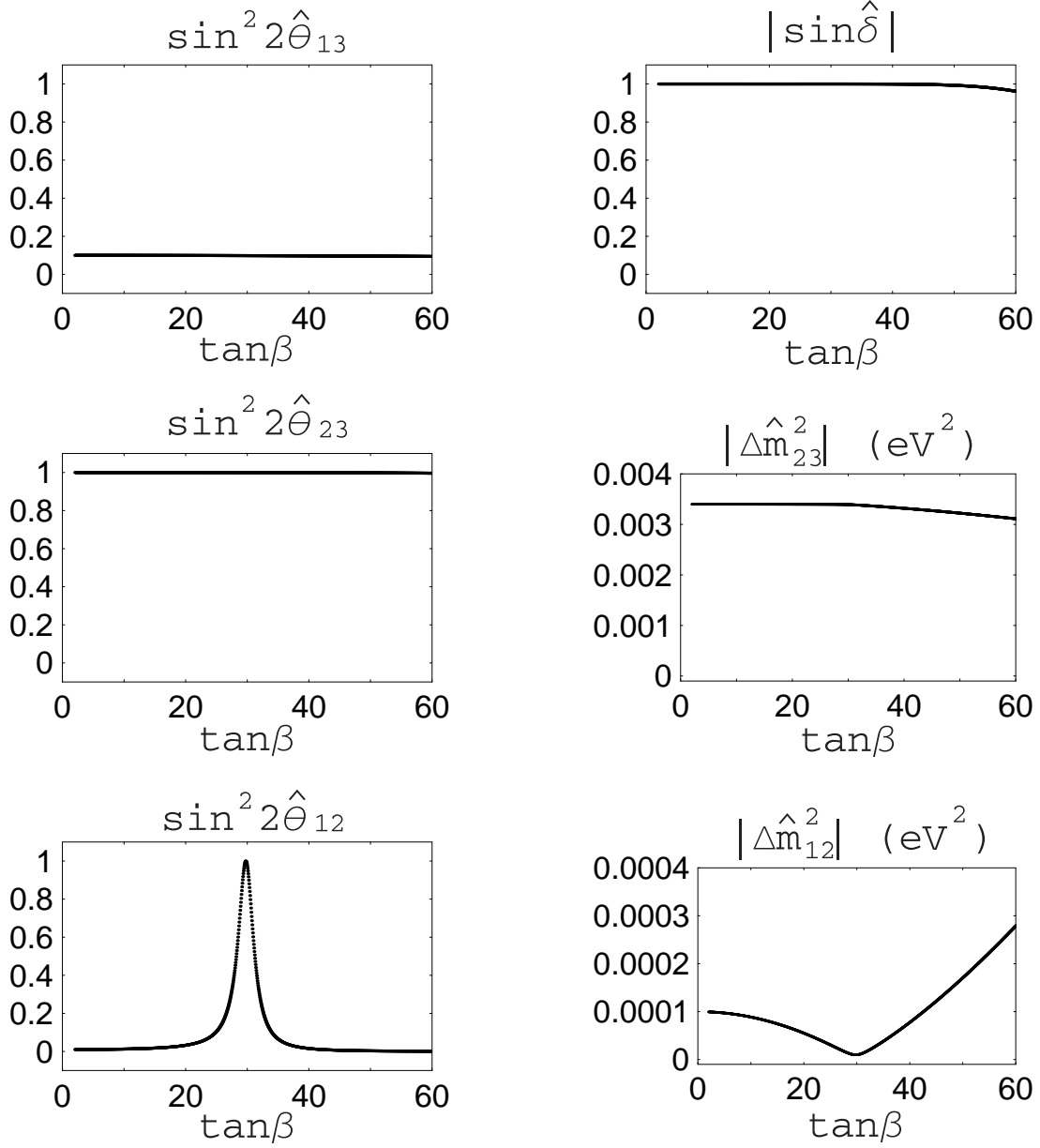


Figure 2: $\tan\beta$ dependence of neutrino mixing angles, Dirac CP phase and mass squared differences at m_Z for $m_3 \ll m_1 < m_2$. As initial values at m_R , we took $(m_1, m_2, m_3) = (5.831 \times 10^{-2}, 5.916 \times 10^{-2}, 0)\text{eV}$, $\sin 2\theta_{12} = \sqrt{8/9}$, $s_{13} = 0.16$ with the model's predictions $\theta_{23} = -\pi/4$ and $\delta = \pi/2$.